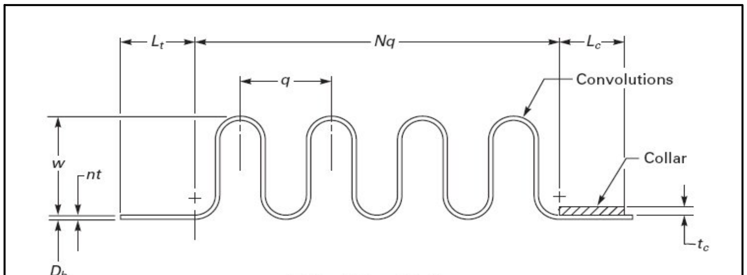
Expansion Joint Analysis for the 1.3-GHz Helium Vessel's Titanium Bellows
29 June 2009

Following Appendix 26 of the ASME BPVC (2007), Section VIII, Division 1



(a) Unreinforced Bellows

DesignPressure (psi)	P := 58
Bellows Inside Diameter (in)	Db := 9.06
Ply thickness (in)	t := 0.012
Number of Plies	n := 1
Bellows Tangent Length (in)	Lt := 0.398
Bellows Mean diameter (in)	Dm := 9.390
Modulus of elasticity (psi)	Eb := 15200000
Convolution height (in)	w := 0.323
Collar length (in)	Lc := 0.170
Collar thickness (in)	tc := 0.186
Collar Modulus of elasticity (psi)	Ec := 15200000
Convolution Pitch (in)	q := 0.341
Kf coefficient 3.0 for as formed bellows 1.5 for annealed bellows	Kf := 3.0
Allowable stress of bellows (psi)	S := 9680
Allowable stress of collar (psi)	Sc := 9680
Weld joint efficiency of collar to bellows	Cwc := 0.6
Number of convolutions	N := 3

Bellows axial stiffness (N/micro-meter)	$Kb := 0.228$
(lbf/inch)	$Kb := Kb \cdot \frac{2.2 \cdot 10^{6} \cdot 2.54}{100}$
	$Kb = 1.274 \times 10^{4}$
Allowable yield stress (psi)	$Sy := 40000$
Poisson's ratio of Ti G2	$\nu b := 0.32$
Bellows live length (inch)	$L := 1.024$
Maximum axial extension (mm)	$x\_positive := 2.1$
(inch)	$x\_positive := \frac{x\_positive}{25.4}$
	$x\_positive = 0.083$
Maximum axial compression (mm)	$x\_negative := 0.33$
(inch)	$x\_negative := \frac{x\_negative}{25.4}$
	$x\_negative = 0.013$

$Dm := Db + w + n \cdot t = 9.395$

$$k := \min\left[\left(\frac{Lt}{1.5\sqrt{Db \cdot t}}\right), 1.0\right] = 0.805$$

$$tp := \left(t\sqrt{\frac{Db}{Dm}}\right) = 0.012$$

$$A := \left[\left(\frac{\pi - 2}{2}\right)q + 2w\right]n \cdot tp = 9.906 \times 10^{-3}$$

$Dc := Db + 2n \cdot t + tc = 9.27$

$$c1 := \frac{q}{2 \cdot w} = 0.528$$

$$c2 := \frac{q}{2.2\sqrt{Dm \cdot tp}} = 0.466$$

$$Ixx := n \cdot tp \cdot \left[\frac{(2 \cdot w - q)^3}{48} + 0.4 \cdot q \cdot (w - 0.2 \cdot q)^2\right]$$

$$e\_eq := \sqrt[3]{12 \cdot \left(1 - \nu b^2\right) \cdot \frac{Ixx}{q}}$$

$D\_eq := Db + w + 2 \cdot e\_eq$

Total axial movement per convolution (inch) $\qquad \Delta q := \frac{(x\_positive + x\_negative)}{N}$

$$
M := \begin{cases}
\begin{pmatrix}
1.001 & -0.448 & -1.244 & 1.932 & -0.398 & -0.291 \\
0.999 & -0.735 & 0.106 & -0.585 & 1.787 & -1.022 \\
0.961 & -1.146 & 3.023 & -7.488 & 8.824 & -3.634 \\
0.955 & -2.708 & 7.279 & 14.212 & -104.242 & 133.333 \\
0.95 & -2.524 & 10.402 & -93.848 & 423.636 & -613.333 \\
0.95 & -2.296 & 1.63 & 16.03 & -113.939 & 240 \\
0.95 & -2.477 & 7.823 & -49.394 & 141.212 & -106.667 \\
0.95 & -2.027 & -5.264 & 48.303 & -139.394 & 160 \\
0.95 & -2.073 & -3.622 & 29.136 & -49.394 & 13.333 \\
0.95 & -2.073 & -3.622 & 29.136 & -49.394 & 13.333 \\
0.95 & -2.073 & -3.622 & 29.136 & -49.394 & 13.333 \\
0.95 & -2.073 & -3.622 & 29.136 & -49.394 & 13.333 \\
0.95 & -2.073 & -3.622 & 29.136 & -49.394 & 13.333
\end{pmatrix} & \text{if } c1 \le 0.3 \\
\begin{pmatrix}
1.001 & -0.448 & -1.244 & 1.932 & -0.398 & -0.291 \\
0.999 & -0.735 & 0.106 & -0.585 & 1.787 & -1.022 \\
0.961 & -1.146 & 3.023 & -7.488 & 8.824 & -3.634 \\
0.622 & 1.685 & -9.347 & 18.447 & -15.991 & 5.119 \\
0.201 & 2.317 & -5.956 & 7.594 & -4.945 & 1.299 \\
0.598 & -0.99 & 3.741 & -6.453 & 5.107 & -1.527 \\
0.473 & -0.029 & -0.015 & -0.03 & 0.016 & 0.016 \\
0.477 & -0.146 & -0.018 & 0.037 & 0.097 & -0.067 \\
0.935 & -3.613 & 9.456 & -13.228 & 9.355 & -2.613 \\
1.575 & -8.646 & 24.368 & -35.239 & 25.313 & -7.157 \\
1.464 & -7.098 & 17.875 & -23.778 & 15.953 & -4.245 \\
1.495 & -6.904 & 16.024 & -19.6 & 12.069 & -2.944 \\
2.037 & -11.037 & 28.276 & -37.655 & 25.213 & -6.716
\end{pmatrix} & \text{if } c1 > 0.3
\end{cases}
$$

$$i := \begin{cases} 0 & \text{if } 0 \le c2 \le 0.2 \\ 1 & \text{if } 0.2 < c2 \le 0.4 \\ 2 & \text{if } 0.4 < c2 \le 0.6 \\ 3 & \text{if } 0.6 < c2 \le 0.8 \\ 4 & \text{if } 0.8 < c2 \le 1.0 \\ 5 & \text{if } 1.0 < c2 \le 1.2 \\ 6 & \text{if } 1.2 < c2 \le 1.4 \\ 7 & \text{if } 1.4 < c2 \le 1.6 \\ 8 & \text{if } 1.6 < c2 \le 2.0 \\ 9 & \text{if } 2.0 < c2 \le 2.5 \\ 10 & \text{if } 2.5 < c2 \le 3.0 \\ 11 & \text{if } 3.0 < c2 \le 3.5 \\ 12 & \text{if } 3.5 < c2 \le 4.0 \end{cases}$$

$$cp := M_{i,0} + M_{i,1}\cdot c1 + M_{i,2}\cdot c1^2 + M_{i,3}\cdot c1^3 + M_{i,4}\cdot c1^4 + M_{i,5}\cdot c1^5 = 0.633$$

$$delt := \begin{cases} 0.2 & \text{if } 0 \le c2 \le 0.2 \\ 0.4 & \text{if } 0.2 < c2 \le 0.4 \\ 0.6 & \text{if } 0.4 < c2 \le 0.6 \\ 0.8 & \text{if } 0.6 < c2 \le 0.8 \\ 1.0 & \text{if } 0.8 < c2 \le 1.0 \\ 1.2 & \text{if } 1.0 < c2 \le 1.2 \\ 1.4 & \text{if } 1.2 < c2 \le 1.4 \\ 1.6 & \text{if } 1.4 < c2 \le 1.6 \\ 2.0 & \text{if } 1.6 < c2 \le 2.0 \\ 2.5 & \text{if } 2.0 < c2 \le 2.5 \\ 3.0 & \text{if } 2.5 < c2 \le 3.0 \\ 3.5 & \text{if } 3.0 < c2 \le 3.5 \\ 4 & \text{if } 3.5 < c2 \le 4.0 \end{cases}$$

$$cp2 := \frac{cp\cdot c2}{delt} = 0.492$$

$$S1 := \frac{(Db + n\cdot t)^2 Lt\cdot Eb\cdot k\cdot P}{2[n\cdot t\cdot (Db + n\cdot t)Lt\cdot Eb + tc\cdot Dc\cdot Lc\cdot Ec\cdot k]} = 2.738 \times 10^3$$

$$S11 := \frac{Dc^2\cdot P\cdot Lt\cdot Ec\cdot k}{2[n\cdot t\cdot (Db + n\cdot t)\cdot Lt\cdot Eb + tc\cdot Dc\cdot Lc\cdot Ec\cdot k]} = 2.859 \times 10^3$$

$$S2e := \frac{P\cdot [q\cdot Dm + Lt\cdot (Db + n\cdot t)]}{2(A + n\cdot tp\cdot Lt + tc\cdot Lc)} = 4.276 \times 10^3$$

$$S2i := \frac{P\cdot q\cdot Dm}{2\cdot A} = 9.379 \times 10^3$$

$$S3 := \frac{P\cdot w}{2n\cdot tp} = 794.884$$

$$S4 := \left(\frac{w}{tp}\right)^2 \cdot \frac{P\cdot cp}{2n} = 1.38 \times 10^4$$

$$Psc := 0.34\cdot \frac{\pi\cdot Kb}{N\cdot q}$$

$$\delta := \frac{S4}{3\cdot S2i}$$

$$\alpha := 1 + 2\cdot \delta^2 + \sqrt{1 - 2\cdot \delta^2 + 4\cdot \delta^4}$$

$$Sy\_eff := 2.3\cdot Sy$$

$$Psi := (\pi - 2)\cdot \frac{A\cdot Sy\_eff}{Dm\cdot q\cdot \sqrt{\alpha}}$$

$\beta 0 := 1.005$ $\beta 1 := 1.9$ $\beta 2 := -3.4$ $\beta 3 := 7$ $\beta 4 := -8.4$ $\beta 5 := 3.37$

$$Cf := \beta 0 + \beta 1\cdot c1 + \beta 2\cdot c1^2 + \beta 3\cdot c1^3 + \beta 4\cdot c1^4 + \beta 5\cdot c1^5$$

$\gamma 0 := 1.0$ $\gamma 1 := 1.7$ $\gamma 2 := -1.14$ $\gamma 3 := 1.75$ $\gamma 4 := 1.75$ $\gamma 5 := 2.1$

$$Cd := \gamma 0 + \gamma 1\cdot c1 + \gamma 2\cdot c1^2 + \gamma 3\cdot c1^3 + \gamma 4\cdot c1^4 + \gamma 5\cdot c1^5$$

$$S5 := \frac{1}{2}\cdot \frac{Eb\cdot tp^2}{w^3\cdot Cf}\cdot \Delta q$$

$$S6 := \frac{5}{3}\cdot \frac{Eb\cdot tp}{w^2\cdot Cd}\cdot \Delta q$$

$$St := 0.7\cdot (S3 + S4) + (S5 + S6)$$

Theoretical axial thickness
(lbf/in)

$$Kb\_theory := \frac{\pi}{2\cdot\left(1 - \nu b^2\right)} \frac{n}{N} \cdot Eb \cdot Dm \cdot \left(\frac{tp}{w}\right)^3 \cdot \frac{1}{Cf}$$

Calculating the buckling pressure for the bellows as an equivalent cylinder

$$\frac{D\_eq}{e\_eq} = 63.704$$

$$\frac{L}{D\_eq} = 0.106$$

$$A\_factor := 0.055$$

$$B\_factor := 19500$$

$$Pa := \frac{4 \cdot B\_factor}{3 \cdot \left(\frac{D\_eq}{e\_eq}\right)}$$

Calculating allowed number of cycles for fatigue (Paragraph 26-6.6.3.2)
(equations are valid for austenitic Cr-Ni SS, UNS N066XX and UNS N04400)

$$Ko := 5.2 \cdot 10^6$$

$$So := 38300$$

$$Eo := Eb$$

$$Kg := 1$$

$$Kg \cdot \frac{Eo}{Eb} \cdot St = 5.516 \times 10^4$$

$$Nalw := \left(\frac{Ko}{Kg \cdot \frac{Eo}{Eb} \cdot St - So}\right)^2$$

circumferential membrane stress in bellows tangent (psi)	$S1 = 2.738 \times 10^3$
circumferential membrane stress in collar (psi)	$S11 = 2.859 \times 10^3$
circumferential membrane stress in bellows (psi) (for end convolution)	$S2e = 4.276 \times 10^3$
circumferential membrane stress in bellows (psi) (for intermediate convolution)	$S2i = 9.379 \times 10^3$
meridional membrane stress in bellow (psi)	$S3 = 794.884$
meridional bending stress in bellows (psi)	$S4 = 1.38 \times 10^4$
allowable internal pressure to avoid column instability (psi)	$Psc = 1.33 \times 10^4$
allowable internal pressure based on in-plane instability (psi)	$Psi = 211.979$
allowable external pressure based on instability (psi)	$Pa = 408.137$
meridional membrane stress (psi)	$S5 = 633.681$
meridional bending stress (psi)	$S6 = 4.432 \times 10^4$
tc tal stress range due to cyclic displacement (psi)	$St = 5.516 \times 10^4$
theoretical axial stiffness (lbf/in)	$Kb\_theory = 2.567 \times 10^3$
allowed number of cycles for fatigue (only valid for materials that are not Titanium)	$Nalw = 9.508 \times 10^4$

ACCEPTANCE CRITERIA

$S1 = 2.738 \times 10^{3}$

$S2e = 4.276 \times 10^{3}$ smaller than ? $S = 9.68 \times 10^{3}$

$S2i = 9.379 \times 10^{3}$

$S11 = 2.859 \times 10^{3}$ $Cwc \cdot Sc = 5.808 \times 10^{3}$

$S3 + S4 = 1.459 \times 10^{4}$ $Kf \cdot S = 2.904 \times 10^{4}$

$P = 58$ $Psc = 1.33 \times 10^{4}$

$Psi = 211.979$

$Pa = 408.137$